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Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2020

Fourth Semester

Faculty of Science

Branch I (A) : Mathematics

MT 04E 04—PROBABILITY THEORY

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Explain axiomatic approach to probability.
2. What are the advantages of classical definition of probability ?
3. Find m.g.f. of gamma distribution with one parameter.
4. Explain the method of maximum likelihood.
5. Briefly explain the three principles of design of experiment.
6. Define Poisson distribution and find its mean.
7. State the assumptions used in analysis of variance.
8. State Lindberg-Levy form of Central Limit theorem.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Prove that negative binomial distribution tends to Poisson distribution under certain condition.
10. Define multinomial distribution. Find its m.g.f.
11. A scientist desires to estimate the mean of a population using a sample sufficiently large such that the probability will be 0.99 that the sample mean will not differ from population mean by more than 25% of the standard deviation. How large the size of the sample should be.
12. Explain moment method of estimation. Estimate the parameters μ and σ^2 of a normal population by the method of moments.

Turn over

13. If T is an unbiased estimator of θ show that T^2 is a biased estimator of θ^2.

14. State and prove Gauss-Markov theorem.

15. Two integers are randomly chosen from the integers 2 to N. What is the probability that they are prime to each other. Find it if N = 10.

16. For the distribution $f(x) = ae^{-ax} a > 0, x > 0$ find the m.g.f. which on expansion gives the central moments. Find the variance of the distribution.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) State and prove Baye's theorem.

    (b) A bag contain 10 balls either black or white. It is not known how many of each color is in the bag. A ball is drawn at random and is found to be white. What is the probability that the bag contain atleast 5 white balls.

18. (a) $\{X_k\}, k = 1,2....$ is a sequence of independent random variables each taking values –1, 0, 1. Given that $P(X_k = 1) = P(X_k = -1) = \frac{1}{k}$ and $P(X_k = 0) = 1 - \frac{2}{k}$. Examine whether the WLLN holds for this sequence.

    (b) Show that means of samples taken from a normal population are consistent estimates of population mean.

19. (a) Derive the m.g.f. of bivariate normal distribution.

    (b) Explain Beta distribution of second kind and find its variance.

20. (a) If X is a random variable distributed as N (0, 1), then show that X^2 has Gamma distribution with the parameters $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$.

    (b) Explain the one-way classification.

21. (a) State and Prove Cramer Rao inequality.

    (b) Obtain the MVB estimator for μ in the normal population $N(\mu, \sigma^2)$ where σ^2 is known.

22. Obtain the most powerful test of size α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ where $\theta_1 > \theta_0$ if the p.d.f. of the population is $f(x,\theta) = \frac{1}{\sqrt{8\pi}} e^{-\frac{1}{8}(x-\theta)^2}, -\infty < x < \infty.$

$(3 \times 5 = 15)$